3rd International Air Transport and Operations Symposium

18 – 20 June 2012
Delft, the Netherlands

Optimal Scheduling of Fuel-Minimal Approach Trajectories

Florian Fisch, Matthias Bittner, Prof. Florian Holzapfel
Institute of Flight System Dynamics, Technische Universität München, Garching, Germany

Outline

1. Introduction
2. Aircraft Simulation Model
3. Multi-Aircraft Optimization Problem
4. Results
5. Summary & Outlook

Outline

Computation of fuel minimal and noise minimal approach trajectories:

So far:

⇨ Optimization of **stand-alone** approach trajectories

⇨ Limitations due to other aircraft in the vicinity of an airport are not taken into account

⇨ Optimization results can not be put into practice due to the limitations arising from the remaining air traffic and the daily airport business

Here:

⇨ **Simultaneous** optimization of the approach trajectories of **multiple** aircraft present in the vicinity of an airport

⇨ Landing sequence is not pre-determined and has to be found by the optimization procedure

⇨ More realistic results

Outline

Point-Mass Simulation Model:

Position Equations of Motion (*NED*-Frame):

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}_O^E = \begin{pmatrix} V_K^G \cdot \cos\chi_K^G \cdot \cos\gamma_K^G \\ V_K^G \cdot \sin\chi_K^G \cdot \cos\gamma_K^G \\ -V_K^G \cdot \sin\gamma_K^G \end{pmatrix}_O$$

Translation Equations of Motion:

$$\begin{pmatrix} \dot{V}_K^G \\ \dot{\chi}_K^G \\ \dot{\gamma}_K^G \end{pmatrix}_K^{EO} = \begin{pmatrix} 1 \\ \dfrac{1}{V_K^G \cdot \cos\gamma_K^G} \\ -\dfrac{1}{V_K^G} \end{pmatrix}_K \left(\frac{1}{m} \left(\sum \vec{\mathbf{F}}^G \right)_K \right)$$

Aircraft Simulation Model

Total sum of external forces:

$$\left(\sum \vec{\mathbf{F}}^G\right)_K = \left(\vec{\mathbf{F}}_A^G\right)_K + \left(\vec{\mathbf{F}}_P^G\right)_K + \mathbf{M}_{KO} \cdot \left(\vec{\mathbf{F}}_G^G\right)_O$$

Thrust modeling:

$$T = \delta_T \cdot T_{\max}$$

$$T_{\max} = T_{\max,ISA} \cdot \left(1 - C_{Tc5} \cdot \Delta T_{ISA,eff}\right)$$

$$T_{\max,ISA} = C_{Tc1} \cdot \left(1 - \frac{h}{C_{Tc2}} + C_{Tc3} \cdot (h)^2\right)$$

$$\Delta T_{ISA,eff} = \Delta T_{ISA} - C_{Tc4}$$

Aerodynamic coefficients:

$$C_D = C_{D0} + C_{D2} \cdot C_L^2$$

$$C_L = C_{L0} + C_{L\alpha} \cdot \alpha_{A,CMD}$$

Aircraft Simulation Model

Aerodynamic Forces:

$$D = \bar{q} \cdot S \cdot C_D = \bar{q} \cdot S \cdot \left(C_{D0} + C_{D2} \cdot C_L^2 \right)$$

$$Q = \bar{q} \cdot S \cdot C_Q = \bar{q} \cdot S \cdot C_{Q\beta} \cdot \beta_A = 0$$

$$L = \bar{q} \cdot S \cdot C_L = \bar{q} \cdot S \cdot \left(C_{L0} + C_{L\alpha} \cdot \alpha_A \right)$$

Dynamic pressure:

$$\bar{q} = 0.5 \cdot \rho \cdot V_A^2$$

Force vector:

$$\left(\vec{\mathbf{F}}^G \right)_A = \left(\vec{\mathbf{F}}_A^G \right)_A + \left(\vec{\mathbf{F}}_P^G \right)_A = \begin{pmatrix} -D \\ 0 \\ -L \end{pmatrix}_A + \begin{pmatrix} T \\ 0 \\ 0 \end{pmatrix}_A = \begin{pmatrix} T - D \\ 0 \\ -L \end{pmatrix}_A$$

Noise model:

Sound pressure level:

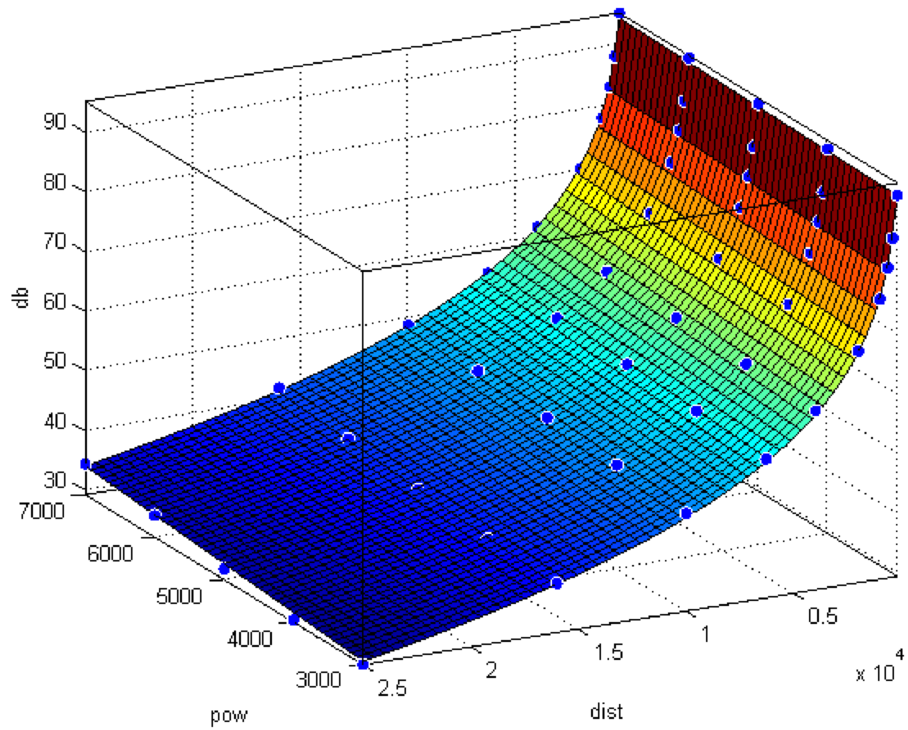
$$L_A(T,r) = a \cdot T + b \cdot \lg(r) + c \cdot \lg(r)^2 + d$$

Sound exposure level:

$$L_{AE} = 10 \lg\left(\frac{1}{t_0}\int_{t_1}^{t_2} 10^{L_A(t)/10}\, dt\right)$$

Number of Awakenings:

$$n_{AW} = \sum_i 0.0087 \cdot \left(L_{AE,i} - 30\right)^{1.79} p_i$$



Atmospheric model (DIN ISO 2533):

$$H_G = \frac{r_E \cdot h}{r_E + h}$$

$$\rho = \rho_S \left[1 + \frac{\gamma_{Tr}}{T_S} \cdot H_G\right]^{\left(-\frac{g_S}{R \cdot \gamma_{Tr}} - 1\right)}$$

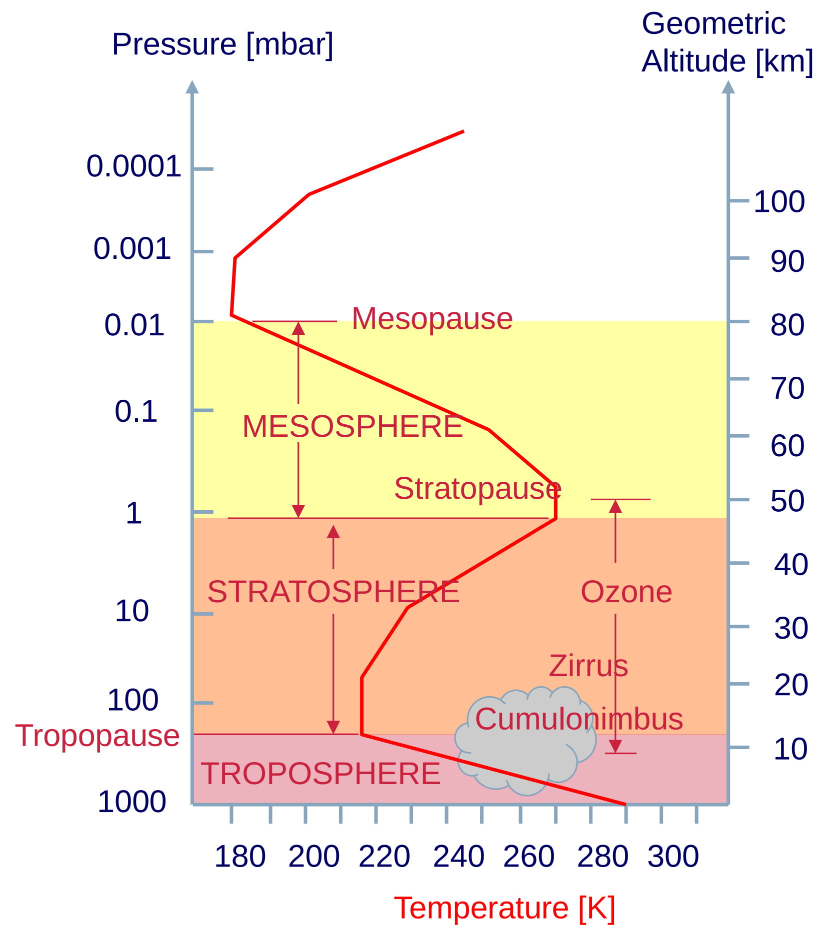
$$p = p_S \left[1 + \frac{\gamma_{Tr}}{T_S} \cdot H_G\right]^{\left(-\frac{g_S}{R \cdot \gamma_{Tr}}\right)}$$

Fuel consumption:

$$\dot{m}_{fuel,idle} = C_{f3} \cdot \left(1 - \frac{h}{C_{f4}}\right)$$

$$\dot{m}_{fuel,\max} = C_{f1} \cdot \left(1 + \frac{V_A}{C_{f2}}\right) \cdot T_{\max}$$

$$\dot{m}_{fuel} = \dot{m}_{fuel,idle} + \delta_T \cdot \left(\dot{m}_{fuel,\max} - \dot{m}_{fuel,idle}\right)$$



Outline

1. Introduction
2. Aircraft Simulation Model
3. **Multi-Aircraft Optimization Problem**
4. Results
5. Summary & Outlook

Multi-Aircraft Optimization Problem

Determine the optimal control histories $\mathbf{u}_{i,opt}(t_i) \in \mathbf{P}^{m}$

and the corresponding optimal state trajectories $\mathbf{x}_{i,opt}(t_i) \in \mathbf{P}^{n}$

that **minimize** the Bolza cost functional

$$J = \sum_{i=1}^{N} \left[e_i\left(\mathbf{x}_i\left(t_{f,i}\right), t_{f,i}\right) + \int_{t_{0,i}}^{t_{f,i}} L_i\left(\mathbf{x}_i(t_i), \mathbf{u}_i(t_i), t_i\right) dt_i \right]$$

subject to

- ⇨ the state dynamics $\dot{\mathbf{x}}_i(t_i) = \mathbf{f}_i\left(\mathbf{x}_i(t_i), \mathbf{u}_i(t_i), t_i\right)$
- ⇨ the initial boundary conditions $\boldsymbol{\psi}_{0,i}\left(\mathbf{x}_i\left(t_{0,i}\right), t_{0,i}\right) = \mathbf{0} \qquad \boldsymbol{\psi}_{0,i} \in \mathbf{P}^{q_i}$
- ⇨ the final boundary conditions $\boldsymbol{\psi}_{f,i}\left(\mathbf{x}_i\left(t_{f,i}\right), t_{f,i}\right) = \mathbf{0} \qquad \boldsymbol{\psi}_{f,i} \in \mathbf{P}^{p_i}$
- ⇨ the interior point conditions $\mathbf{r}_i\left(\mathbf{x}(t_i), t_i\right) = \mathbf{0} \qquad \mathbf{r}_i \in \mathbf{P}^{k_i}$
- ⇨ the equality constraints $\mathbf{C}_{eq,i}\left(\mathbf{x}_i(t_i), \mathbf{u}_i(t_i), t_i\right) = \mathbf{0} \qquad \mathbf{C}_{eq,i} \in \mathbf{P}^{r_i}$
- ⇨ and the inequality constraints $\mathbf{C}_{ineq,i}\left(\mathbf{x}_i(t_i), \mathbf{u}_i(t_i), t_i\right) \leq \mathbf{0} \qquad \mathbf{C}_{ineq,i} \in \mathbf{P}^{s_i}$

$$i = 1, \ldots, N$$

Multi-Aircraft Optimization Problem

Initial boundary conditions:

⇨ Defined by the entry position into the considered air space

Final boundary conditions:

⇨ Assure that the aircraft are finally located on the ILS glide path

⇨ Final approach fix: located at the origin of the Local Fixed Frame N at an altitude of h_{FAF}

⇨ ILS glide path: directed parallel to the x-axis of the Local Fixed Frame N, into the direction of the positive x-axis

Multi-Aircraft Optimization Problem

Final boundary conditions:

- ⇨ Northward position: $x\left(t_f\right) \geq x_{FAF} + \Delta x$
- ⇨ Eastward position: $y\left(t_f\right) = y_{FAF}$
- ⇨ Altitude: $h\left(t_f\right) = h_{FAF} + \tan\left(-\gamma_{K,ILS}\right) \cdot x\left(t_f\right)$
- ⇨ Glide-path angle: $\gamma_K\left(t_f\right) = \gamma_{K,ILS}$
- ⇨ Heading angle: $\chi_K\left(t_f\right) = \chi_{K,ILS}$
- ⇨ Kinematic velocity: $V_K\left(t_f\right) = V_{K,ILS}$

Multi-Aircraft Optimization Problem

Inequality path constraints:

⇨ Load factor: $n_{Z,LB} = 0.85 \le n_Z(t) \le 1.15 = n_{Z,UB}$

⇨ Kinematic velocity: $V_{K,LB} = 200\,\frac{km}{h} \le V_K(t) \le 1000\,\frac{km}{h} = V_{K,UB}$

⇨ Angle of attack: $\alpha_{A,CMD,LB} = -5.73° \le \alpha_{A,CMD} \le 20.05° = \alpha_{A,CMD,UB}$

⇨ Bank angle: $\mu_{K,CMD,LB} = -45° \le \mu_{K,CMD} \le 45° = \mu_{K,CMD,UB}$

⇨ Thrust lever: $\delta_{T,CMD,LB} = 0.0 \le \delta_{T,CMD} \le 1.0 = \delta_{T,CMD,UB}$

⇨ Eastward position: $y_{LB}(t) \le y(t) \le y_{UB}(t)$

⇨ Altitude: $h_{LB}(t) \le h(t) \le h_{UB}(t)$

⇨ Aircraft distance: $d_{ij}(t) - d_{\min} \ge 0$

Inequality path constraints:

- ⇨ Path constraints are formulated such that the aircraft have to follow the ILS glide path once the FAF has been passed

- ⇨ Eastward position:

$$y_{LB}(t) = 0.5 \cdot [1 - \tanh(a \cdot x(t))] \cdot y_{LB} - 100.0$$
$$y_{UB}(t) = 0.5 \cdot [1 - \tanh(a \cdot x(t))] \cdot y_{UB} + 100.0$$

- ⇨ Altitude:

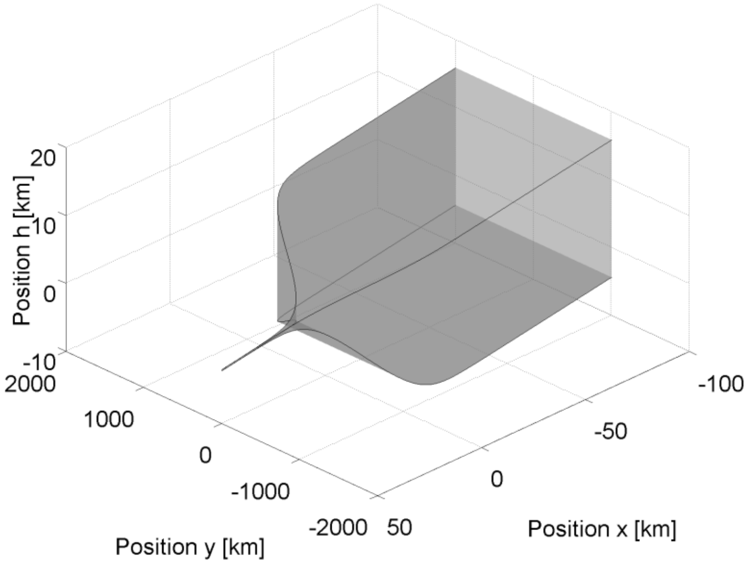
$$h_{LB}(t) = 0.5 \cdot [1 - \tanh(a \cdot x(t))] \cdot h_{LB} + (h_{FAF} - 100.0) + \tan(-\gamma_{ILS}) \cdot x(t)$$
$$h_{UB}(t) = 0.5 \cdot [1 - \tanh(a \cdot x(t))] \cdot h_{UB} + (h_{FAF} + 100.0) + \tan(-\gamma_{ILS}) \cdot x(t)$$

- ⇨ Kinematic velocity:

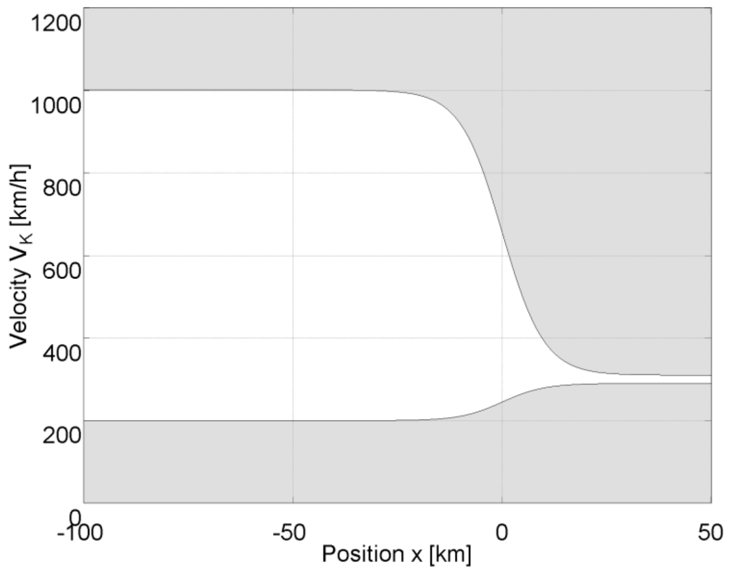
$$V_{K,LB}(t) = 0.5 \cdot [1 - \tanh(a \cdot x(t))] \cdot (V_{K,LB} - V_{K,ILS} + 10.0) + (V_{K,ILS} - 10.0)$$
$$V_{K,UB}(t) = 0.5 \cdot [1 - \tanh(a \cdot x(t))] \cdot (V_{K,UB} - V_{K,ILS} - 10.0) + (V_{K,ILS} + 10.0)$$

Inequality path constraints:

⇨ Path constraints are formulated such that the aircraft have to follow the ILS glide path once the FAF has been passed



Permitted airspace



Path constraints w.r.t. kinematic velocity V_K

Multi-Aircraft Optimization Problem

Inequality path constraints:

⇨ A certain separation distance between the aircraft has to be maintained

Aircraft distances:

$$d_{ij}(t)=\sqrt{\left[x_i(t)-x_j(t)\right]^2+\left[y_i(t)-y_j(t)\right]^2+\left[z_i(t)-z_j(t)\right]^2}, \qquad i=1,\ldots,N,\ j=i+1,\ldots,N$$

Normalization of flight times w.r.t. final flight times:

$$\tau_i=\frac{t_i}{t_{f,i}}, i=1,\ldots,N$$

Introduction of **one** single parameter for all flight times:

$$t_f=t_{f,1}=t_{f,2}=\ldots=t_{f,N}$$

⇨ The time elapsed is the **same** for all aircraft

⇨ Constraints w.r.t. the minimum distances can be checked directly (because of the direct correlation of the time elapsed)

Multi-Aircraft Optimization Problem

Cost function:

Fuel-minimal approaches: maximize aircraft masses at the final times

$$J = -\sum_{i=1}^{N} m_i\left(t_{f,i}\right)$$

Noise-minimal approaches: minimize maximum sound pressure level or number of awakenings

Integral cost functions:

⇨ The same flight time for all aircraft is enforced

⇨ Aircraft are located on different positions on the ILS glide path (i.e. they have covered different distances)

⇨ **Equal weighting** of the aircraft has to be achieved

Portion of the integral cost function originating from flight along ILS glide path is not incorporated into the integral cost function:

$$\dot{m}_{fuel,eff}(t) = \dot{m}_{fuel}(t) \cdot 0.5 \cdot \left[1 - \tanh\left(a \cdot \left[x(t) - \Delta x_{fuel}\right]\right)\right]$$

Full-Discretization Method – Forward (explicit) Euler:

⇨ Time discretization (e.g. equidistant):

$$\tau_i = t_0 + (i-1)\cdot h, \quad i = 1,\ldots,N, \qquad h = \frac{t_f - t_0}{N-1}$$

⇨ Discretization of controls and states at time discretization points:

$$\mathbf{x}_i, \mathbf{u}_i, \qquad i = 1,\ldots,N$$

⇨ Approximation of differential equations:

$$\mathbf{x}_{i+1} = \mathbf{x}_i + h\cdot\mathbf{f}(\mathbf{x}_i, \mathbf{u}_i, \mathbf{p}), \qquad i = 1,\ldots,N-1$$

⇨ Additional equality constraints:

$$\mathbf{x}_{i+1} - \mathbf{x}_i - h\cdot\mathbf{f}(\mathbf{x}_i, \mathbf{u}_i, \mathbf{p}) = \mathbf{0}, \qquad i = 1,\ldots,N-1$$

Multi-Aircraft Optimization Problem

Discretized Optimal Control Problem (Euler):

Determine the optimal parameter vector $\mathbf{z} = [\mathbf{x}, \mathbf{u}]^T$

that minimizes the cost function $J(\mathbf{z})$

subject to

⇨ the inequality constraints $\mathbf{C}_{ineq}(\mathbf{x}(\mathbf{z}), \mathbf{z}) \leq \mathbf{0}$

⇨ and the equality constraints

$$\mathbf{C} = \begin{pmatrix} \boldsymbol{\psi}_0(\mathbf{x}(\mathbf{z}), \mathbf{z}) \\ \mathbf{C}_{eq}(\mathbf{x}(\mathbf{z}), \mathbf{z}) \\ \mathbf{x}_{i+1} - \mathbf{x}_i - h \cdot \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i) \\ \mathbf{r}(\mathbf{x}(\mathbf{z}), \mathbf{z}) \\ \boldsymbol{\psi}_f(\mathbf{x}(\mathbf{z}), \mathbf{z}) \end{pmatrix} = \mathbf{0}$$

⇨ **SNOPT** (sequential quadratic programming SQP)

Solution Strategy:

(1) Optimization without distance path constraints

(2) Simultaneous optimization with distance path constraints, using previous results as initial guess

- ⇨ Distance path constraints fulfilled by initial guess: Initial guess = Optimal solution of constrained problem
- ⇨ Distance path constraints **not** fulfilled by initial guess: Separation of aircraft until path constraints are met

Assumptions:

- ⇨ Optimal solution of unconstrained problem = Excellent initial guess of constrained problem
- ⇨ Cost function of a specific optimization problem is always **less than or equal** to the cost function of the same trajectory optimization problem with additional constraints

Outline

1. Introduction
2. Aircraft Simulation Model
3. Multi-Aircraft Optimization Problem
4. **Results**
5. Summary & Outlook

Results

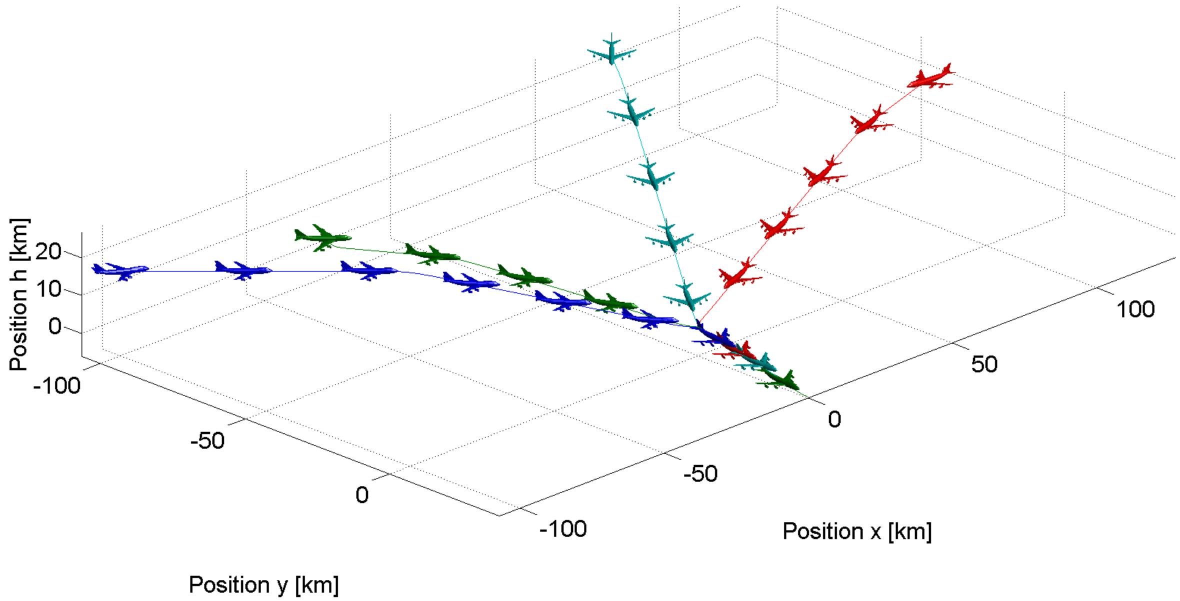
Generic scenario:

⇨ The optimal landing sequence and the optimal approach trajectories for **four** aircraft are sought

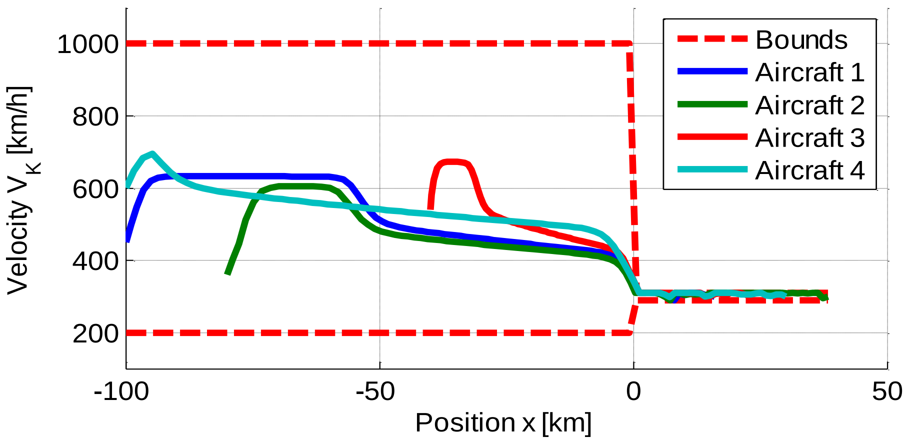
⇨ Initial conditions:

AC No.	1	2	3	4
$x_{i,0}$	$-100000\ m$	$-80000\ m$	$-40000\ m$	$-100000\ m$
$y_{i,0}$	$100000\ m$	$50000\ m$	$-120000\ m$	$-70000\ m$
$h_{i,0}$	$4000\ m$	$4000\ m$	$5000\ m$	$6000\ m$
$\chi_{K,i,0}$	$-60.0°$	$-45.0°$	$95.0°$	$45.0°$
$\gamma_{K,i,0}$	$0.0°$	$0.0°$	$0.0°$	$0.0°$
$V_{K,i,0}$	$450.0\ km/h$	$360.0\ km/h$	$540.0\ km/h$	$600.0\ km/h$

Results

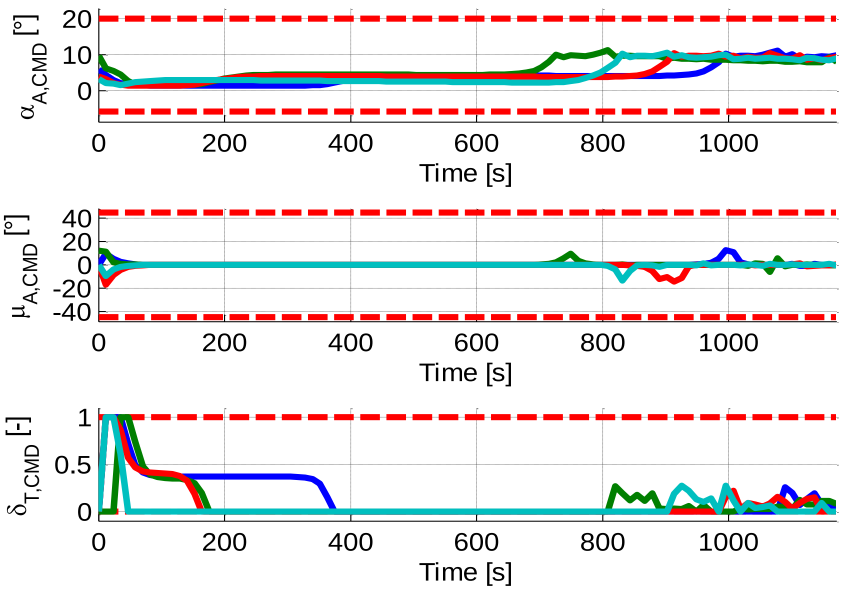


Optimized approach trajectories

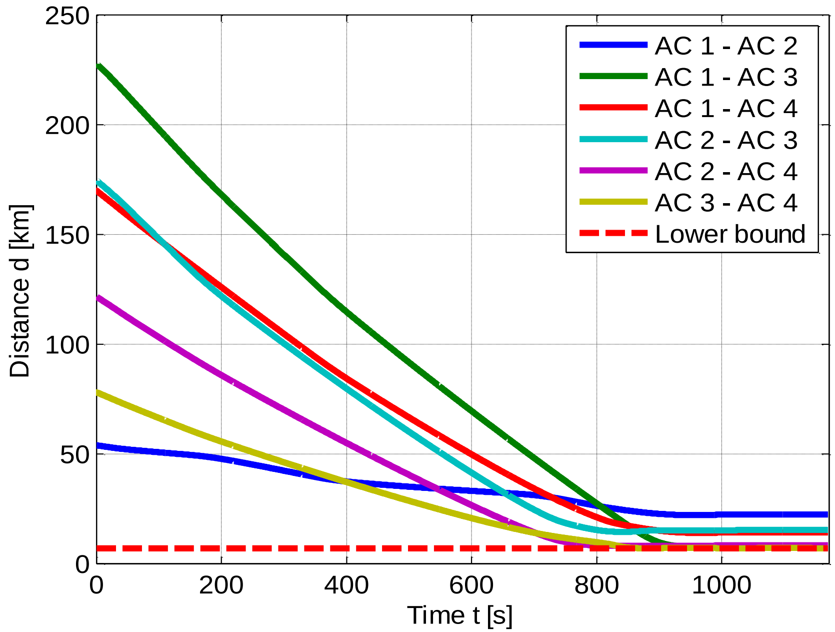


Optimized aircraft velocities V_K

Results



Optimized time histories of aircraft controls



Distances between aircraft

Outline

1. Introduction
2. Aircraft Simulation Model
3. Multi-Aircraft Optimization Problem
4. Results
5. **Summary & Outlook**

Summary

⇨ The approach trajectories of multiple aircraft in the vicinity of an airport have been optimized **simultaneously**

⇨ Path constraints have been introduced so that the aircraft are finally located on the ILS glide path and keep a certain separation distance

⇨ The optimal landing sequence is determined by the optimization procedure

Outlook

⇨ Utilization of splines to describe the centerlines of the allowed flight path corridors for the involved aircraft

⇨ Introduction of path constraints w.r.t. the maximum lateral and horizontal deviation from the centerlines

⇨ More sophisticated distance path constraints between aircraft

Thank you very much for your attention!